Koordinatentransformationen 3x3

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Aufgabe 1

$\mathbb{F} = \left(\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} ; \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right), \quad {}_{\mathbb{F}}P = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

a) ${}_{\mathbb{E}}P = ?$

${}_{\mathbb{E}}P = F \, {}_{\mathbb{F}}P + f$

$f = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \quad F = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

$\Rightarrow {}_{\mathbb{E}}P = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 2 \\ 5 \end{pmatrix}$

b) ${}_{\mathbb{E}}K_{\mathbb{F}}(v) = ?$

${}_{\mathbb{E}}K_{\mathbb{F}}(v) = Fv + f, \quad v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$\Rightarrow {}_{\mathbb{E}}K_{\mathbb{F}}(v) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} v + \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

c) $F^{-1} = ?$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[z_2 - z_3]{z_1 - z_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\Rightarrow F^{-1} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

d) ${}_{\mathbb{F}}K_{\mathbb{E}} = F^{-1}(v - f) = F^{-1}v - F^{-1}f$

$= \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} v - \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} v - \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix}$

Aufgabe 2

$\mathbb{F} = \left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} ; \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right), \quad {}_{\mathbb{F}}P = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

a) ${}_{\mathbb{E}}P = ?$

$F = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \quad f = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

${}_{\mathbb{E}}P = F \, {}_{\mathbb{F}}P + f$

$\Rightarrow {}_{\mathbb{E}}P = \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}$

b) ${}_{\mathbb{E}}K_{\mathbb{F}}(v) = Fv + f$

$= \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} v + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

c) ${}_{\mathbb{F}}K_{\mathbb{E}}(v) = ?$

${}_{\mathbb{F}}K_{\mathbb{E}}(v) = F^{-1}(v - f) = F^{-1}v - F^{-1}f$

$= \frac{1}{2} \begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} v - \frac{1}{2} \begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$= \frac{1}{2} \begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} v - \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$